DISCUSSION OF A TRILEMMA FOR ASSET DEMAND ESTIMATION

William Fuchs, Satoshi Fukuda, Daniel Neuhann Discussed by Julie Zhiyu Fu

OVERVIEW

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- Trilemma of estimation of asset demand
 - Prices respect no-arbitrage,
 - Investors care about asset payoffs,
 - Asset-level demand elasticities can be recovered from supply shocks
- If all three hold, we would have some peculiar asset payoff structure

- The growing demand literature has been heavily empirical, and would benefit from theoretical inputs
- Trilemma of estimation of asset demand, under a restrictive setting
 - Prices respect no-arbitrage,
 - Investors care about asset payoffs, ...in a representative-agent endowment economy with restrictions on utilities
 - Asset-level demand elasticities can be recovered from supply shocks
 There is no spillover effect of supply shocks
- If all three hold, we would have some peculiar asset payoff structure
- However, my discussion:
 - The setting in this paper is restrictive and less relevant for empirical work
 - A counterexample to illustrate the restrictions
 - An empiricist's perspective on demand elasticity estimation

TRILEMMA CONDITION 1: "PRICES RESPECT NO-ARBITRAGE"

Notation: bold symbols for vectors and matrices **p**: asset prices; **q**: state prices; **Y**: asset \times state payoffs; *j* indexes assets; *z* indexes states

TRILEMMA CONDITION 1: "PRICES RESPECT NO-ARBITRAGE"

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 p: asset prices; q: state prices; Y: asset × state payoffs;
 j indexes assets; z indexes states
- No-arbitrage \implies There exists (at least one set of) state prices q such that

$$p = Yq \implies q = Y^{-1}p$$

where, with abuse of notation, \mathbf{Y}^{-1} is the Moore–Penrose pseudoinverse of \mathbf{Y} .

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■ The ideal variation: given an exogenous variation in **p**, the variation in **q** is given as:

$$\Delta oldsymbol{q}^{ideal} \equiv rac{\partial oldsymbol{q}}{\partial oldsymbol{p}^ op} = oldsymbol{Y}^{-1}$$

TRILEMMA CONDITION 2: "INVESTORS CARE ABOUT ASSET PAYOFFS"

Definition (Downward-sloping consumption demand)

- Let *E* be the vector of aggregate asset endowments
- The aggregate consumption endowment in each state is $\mathbf{D} = \mathbf{Y}^{\top} \mathbf{E}$.
- An economy has downward-sloping consumption demand if

$$rac{\partial \mathbf{q}}{\partial \mathbf{E}^{ op}} \equiv -\mathbf{V}\mathbf{Y}^{ op},$$

where $\mathbf{V} \equiv -\frac{\partial \mathbf{q}}{\partial \mathbf{p}^{\top}}$ is a strictly positive diagonal matrix.

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- State-price responses to a one-unit decrease in aggregate consumption:
- Standard representative-agent endowment economy:

$$q_z \equiv \beta \pi_z \frac{u'(C_z)}{u'(C_0)} = \beta \pi_z \frac{u'(D_z)}{u'(D_0)} \qquad -\frac{\partial q_z}{\partial D_{z'}} = \begin{cases} \beta \pi_z \frac{-u''(C_z)}{u'(C_0)} & z' = z \\ 0 & z' \neq z \end{cases}$$

Diagonal V: an increase in endowment in state z only affects the state price in z:

$$q_{z'} = \beta \pi_{z'} \frac{u'(\mathsf{C}_{z'})}{u'(\mathsf{C}_0)}$$

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A crucial assumption for the proof, but quite restrictive!

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■ Investors can reoptimize across states: $\frac{\partial u'(C_0)}{\partial D_z} \neq 0, \frac{\partial u'(C_{z'})}{\partial D_z} \neq 0$

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 - Representative agent: $C_{i,z} = D_z$ for every investor
 - Time- and state-separable utility: $V'(C_z)$ does not depend on $C_{z'}$ (not true in recursive utility)
- Not a typical environment in which demand-based asset pricing is studied
- A counterexample later with non-diagonal V

TRILEMMA CONDITION 3: "RECOVER ELASTICITIES FROM SUPPLY SHOCKS"

Definition (Identical variation)

The ideal state price variation for asset j can be generated by a supply shock to asset j if there exists some scalar k_i such that:

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}_{j}} \times k_{j} = \frac{\partial \mathbf{q}}{\partial \mathbf{E}_{j}} \tag{1}$$

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(1)

The trilemma: Recall

- No arbitrage: $\mathbf{p} = \mathbf{Y}\mathbf{q} \implies \frac{\partial \mathbf{q}}{\partial \mathbf{p}^{\top}} = \mathbf{Y}^{-1}$
- **?** Downward-sloping consumption demand: $\frac{\partial \mathbf{q}}{\partial \mathbf{E}^{\top}} = -\mathbf{V}\mathbf{Y}^{\top}$

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Clarification on Restrictions

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⇒ No two assets can pay off in the same state: A trilemma!

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But why is this condition important for "recovering elasticities from supply shocks?"

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Trilemma, restated: A supply shock has no spillover effect only if assets have no overlapping payoffs (under diagonal **V**)

- Seems right...but also not surprising?
- But why is the no-spillover condition necessary for asset elasticity estimation?

How is "asset price elasticity" defined? It is not elaborated in the paper. My take:

Under no spillover, asset elasticity
$$= \frac{1}{price impact}$$

Let a(p) denote the portfolio choice. We define $\Gamma \equiv \frac{\partial a}{\partial p^{\top}}$ as the elasticity matrix

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- Market clearing gives the price response to supply:

$$a(\mathbf{p}) = \mathbf{E} \implies \Delta \mathbf{p}^{supply} \equiv \frac{\partial \mathbf{p}}{\partial \mathbf{E}^{\top}} = \Gamma^{-1}$$

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- If we want the inverse of price impact $\frac{1}{(\Gamma^{-1})_{ii}} = \Gamma_{jj}$ elasticity, we need diagonal Γ^{-1}
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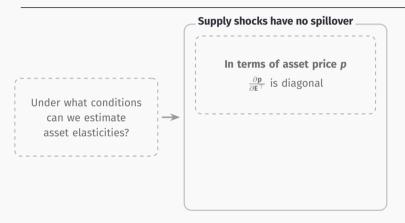
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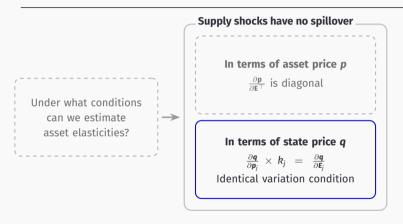
But that's not how empirical literature estimates elasticities!

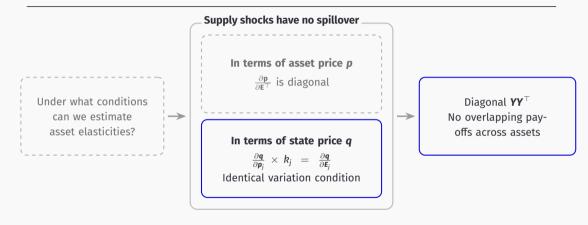
TRILEMMA, THE LOGIC CHAIN

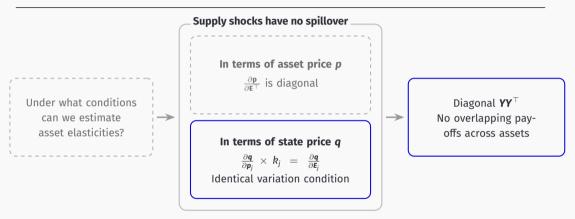
Under what conditions can we estimate asset elasticities?



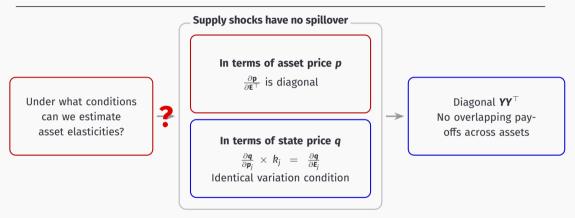
A Counterexample







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- But asset demand and prices should not be swept under the rug
 - The link is also not justified

QUICK COMMENT ON SAME-SIGN CONDITION

Same-sign condition: $-\frac{\partial \mathbf{q}}{\partial \mathbf{E}^{\top}} \equiv \mathbf{V}\mathbf{Y}^{\top}$ and $\frac{\partial \mathbf{q}}{\partial \mathbf{p}^{\top}} \equiv \mathbf{Y}^{-1}$ have the same sign **With diagonal and positive V**: \mathbf{Y}^{-1} is non-negative for all entries $\Longrightarrow \mathbf{Y}\mathbf{Y}^{\top}$ is diagonal Two comments:

- Without diagonal and positive V, less can be said about the sign of Y^{-1} .
- $oldsymbol{\Phi}$ Even under **V**, it is unclear why this is a crucial for estimating elasticity $rac{\partial oldsymbol{a}}{\partial oldsymbol{p}^{-1}}$
 - What's wrong with an increase in an asset's price leading to a decrease in certain state prices?
 - There might be deeper reasons, but they need to be spelled out

A COUNTEREXAMPLE: ELASTICITY IDENTIFICATION UNDER CARA

Representative agent, static CARA, with two assets with payoffs X_1, X_2 :

$$\begin{split} \max_{a_1,a_2} & \mathbb{E}\left[-e^{-\gamma W}\right] \qquad W = W_0 - \sum_{j=1}^2 p_j a_j + \sum_{j=1}^2 a_j X_j \\ \text{where } X_j = F + \epsilon_j \quad F \sim \mathcal{N}(\mu_F, \sigma_F^2), \quad \epsilon_j \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad F \perp \epsilon_1 \perp \epsilon_2 \end{split}$$

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The solution to the portfolio choice is given as:

$$a_1^{\star} = \frac{1}{\gamma(2\sigma_F^2 + \sigma_\epsilon^2)} \left(\mu_F - p_1\right) + \frac{\sigma_F^2}{\gamma\sigma_\epsilon^2(2\sigma_F^2 + \sigma_\epsilon^2)} \left(p_2 - p_1\right)$$

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■ Imposing market clearing $a_i^* = E_i$ and solving for prices yields:

$$p_1 = \mu_F - \gamma \left[\sigma_{\epsilon}^2 E_1 + \sigma_F^2 (E_1 + E_2) \right].$$

Asset 2 is symmetric.

WHEN THERE IS NO SPILLOVER

- Consider the case where $\sigma_F^2 = 0$, so there is no spillover effect
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- lacktriangle Demand for asset 1 does not depend on the price of asset 2 \Longrightarrow no spillover effect
- We can estimate the demand elasticity using the inverse of the price impact:

$$\frac{1}{\partial p_j/\partial E_j} = -\frac{1}{\gamma \sigma_{\epsilon}^2}$$

- lacktriangle As payoffs are independent, they overlap in all states! (YY^{\top} is not diagonal)
- lacktriangle Why doesn't the trilemma apply? $oldsymbol{V}$ is not diagonal in this economy

Now consider the case where $\sigma_F^2 > 0$, so there is a spillover effect

$$p_1 = \mu_F - \gamma \left[\sigma_\epsilon^2 E_1 + \sigma_F^2 (E_1 + E_2) \right]$$

• Now consider the case where $\sigma_{\rm F}^2 > 0$, so there is a spillover effect

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■ The inverse of price impact no longer equals the elasticity

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- An analogy from factor models:
 - An impossibility theorem: We can't solve portfolio choice problems because we can't accurately estimate an $N \times N$ covariance matrix Σ

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 - Advantage: Flexible and model-agnostic
 - Disadvantage: Too much flexibility! Need more theory-guided structures
- If a more structural model can fit better, just estimate the structural parameters
 - Current state: relatively few models can be easily taken to data
 - Theoretical contributions are highly valued!
- Huge synergy between theories and empirics. Looking forward to more theoretical discussions!

