Discussion of A Trilemma for Asset Demand Estimation

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Discussed by Julie Zhiyu Fu

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- Trilemma of estimation of asset demand
  1. Prices respect no-arbitrage,
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  3. Asset-level demand elasticities can be recovered from supply shocks
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- The growing demand literature has been heavily empirical, and would benefit from theoretical inputs

- Trilemma of estimation of asset demand, under a restrictive setting
  1. Prices respect no-arbitrage,
  2. Investors care about asset payoffs, ...in a representative-agent endowment economy with restrictions on utilities
  3. ~~Asset-level demand elasticities can be recovered from supply shocks~~ There is no spillover effect of supply shocks
- If all three hold, we would have some peculiar asset payoff structure
- However, my discussion:
  - The setting in this paper is restrictive and less relevant for empirical work
  - A counterexample to illustrate the restrictions
  - An empiricist's perspective on demand elasticity estimation

Trilemma Condition 1: "Prices respect no-arbitrage"

- Notation: bold symbols for vectors and matrices
  $\boldsymbol{p}$: asset prices; $\boldsymbol{q}$: state prices; $\boldsymbol{Y}$: asset $\times$ state payoffs;
  j indexes assets; z indexes states

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  j indexes assets; z indexes states
- No-arbitrage $\implies$ There exists (at least one set of) state prices $\boldsymbol{q}$ such that

$$\boldsymbol{p} = \boldsymbol{Y}\boldsymbol{q} \quad \implies \quad \boldsymbol{q} = \boldsymbol{Y}^{-1}\boldsymbol{p}$$

where, with abuse of notation, $\boldsymbol{Y}^{-1}$ is the Moore–Penrose pseudoinverse of $\boldsymbol{Y}$.

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  where, with abuse of notation, $\boldsymbol{Y}^{-1}$ is the Moore–Penrose pseudoinverse of $\boldsymbol{Y}$.
- The ideal variation: given an exogenous variation in $\boldsymbol{p}$, the variation in $\boldsymbol{q}$ is given as:

$$\Delta \boldsymbol{q}^{ideal} \equiv \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{p}^\top} = \boldsymbol{Y}^{-1}$$

Trilemma Condition 2: "Investors care about asset payoffs"

Definition (Downward-sloping consumption demand)

- Let $\boldsymbol{E}$ be the vector of aggregate asset endowments
- The aggregate consumption endowment in each state is $\boldsymbol{D} = \boldsymbol{Y}^\top \boldsymbol{E}$.
- An economy has downward-sloping consumption demand if

$$\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{E}^\top} \equiv -\boldsymbol{V}\boldsymbol{Y}^\top,$$

where $\boldsymbol{V} \equiv -\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{D}^\top}$ is a strictly positive diagonal matrix.

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  - State-price responses to a one-unit decrease in aggregate consumption:
  - Standard representative-agent endowment economy:

$$q_z \equiv \beta\pi_z \frac{u'(C_z)}{u'(C_0)} = \beta\pi_z \frac{u'(D_z)}{u'(D_0)} \qquad -\frac{\partial q_z}{\partial D_{z'}} = \begin{cases} \beta\pi_z \frac{-u''(C_z)}{u'(C_0)} & z' = z \\ 0 & z' \neq z \end{cases}$$

Trilemma Condition 2: The Diagonal V Assumption

Diagonal $\boldsymbol{V}$: an increase in endowment in state z only affects the state price in z:

A crucial assumption for the proof, but quite restrictive!

$$q_{z'} = \beta \pi_{z'} \frac{u'(C_{z'})}{u'(C_0)}$$

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  - Representative agent: $C_{i,z} = D_z$ for every investor
  - Time- and state-separable utility: $V'(C_z)$ does not depend on $C_{z'}$ (not true in recursive utility)
- Not a typical environment in which demand-based asset pricing is studied
- A counterexample later with non-diagonal **V**

Trilemma Condition 3: "Recover elasticities from Supply Shocks"

Definition (Identical variation)
The ideal state price variation for asset j can be generated by a supply shock to asset j if there exists some scalar k_j such that:

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But why is this condition important for "recovering elasticities from supply shocks?"

What Exactly Does "Identical Variation" Mean?

The identical variation condition :

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- But why is the no-spillover condition necessary for asset elasticity estimation?

WHY WE "NEED" NO SPILLOVER FOR RECOVERING ELASTICITIES?

How is "asset price elasticity" defined? It is not elaborated in the paper. My take:

Under no spillover, asset elasticity $= \frac{1}{\textbf{price impact}}$

- Let $\boldsymbol{a}(\boldsymbol{p})$ denote the portfolio choice. We define $\Gamma \equiv \frac{\partial \boldsymbol{a}}{\partial \boldsymbol{p}^\top}$ as the elasticity matrix

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- If we want the inverse of price impact $\frac{1}{(\Gamma^{-1})_{jj}} = \Gamma_{jj}$ elasticity, we need diagonal $\boldsymbol{\Gamma}^{-1}$
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But that's not how empirical literature estimates elasticities!

Trilemma, the Logic Chain

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In terms of asset price *p*

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- The paper mainly discusses the link between state prices and state payoffs
- But asset demand and prices should not be swept under the rug
  - 1 The link is also not justified

Quick Comment on Same-sign Condition

Same-sign condition: $-\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{E}^\top} \equiv \boldsymbol{V}\boldsymbol{Y}^\top$ and $\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{p}^\top} \equiv \boldsymbol{Y}^{-1}$ have the same sign

With diagonal and positive V: $\boldsymbol{Y}^{-1}$ is non-negative for all entries $\implies$ $\boldsymbol{Y}\boldsymbol{Y}^\top$ is diagonal

Two comments:

1. Without diagonal and positive $\boldsymbol{V}$, less can be said about the sign of $\boldsymbol{Y}^{-1}$.
2. Even under $\boldsymbol{V}$, it is unclear why this is a crucial for estimating elasticity $\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{p}^\top}$
   - What's wrong with an increase in an asset's price leading to a decrease in certain state prices?
   - There might be deeper reasons, but they need to be spelled out

A Counterexample: Elasticity Identification under CARA

- Representative agent, static CARA, with two assets with payoffs X_1, X_2:

$$\max_{a_1,a_2} \mathbb{E}\left[-e^{-\gamma W}\right] \qquad W = W_0 - \sum_{j=1}^{2} p_j a_j + \sum_{j=1}^{2} a_j X_j$$

$$\text{where } X_j = F + \epsilon_j \quad F \sim \mathcal{N}(\mu_F, \sigma_F^2), \quad \epsilon_j \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad F \perp \epsilon_1 \perp \epsilon_2$$

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- The solution to the portfolio choice is given as:

$$a_1^\star = \frac{1}{\gamma(2\sigma_F^2 + \sigma_\epsilon^2)}(\mu_F - p_1) + \frac{\sigma_F^2}{\gamma\sigma_\epsilon^2(2\sigma_F^2 + \sigma_\epsilon^2)}(p_2 - p_1)$$

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$$a_1^\star = \frac{1}{\gamma(2\sigma_F^2 + \sigma_\epsilon^2)}(\mu_F - p_1) + \frac{\sigma_F^2}{\gamma\sigma_\epsilon^2(2\sigma_F^2 + \sigma_\epsilon^2)}(p_2 - p_1)$$

- Imposing market clearing $a_j^\star = E_j$ and solving for prices yields:

$$p_1 = \mu_F - \gamma\left[\sigma_\epsilon^2 E_1 + \sigma_F^2(E_1 + E_2)\right].$$

Asset 2 is symmetric.

When There is No Spillover

- Consider the case where $\sigma_F^2 = 0$, so there is no spillover effect
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- We can estimate the demand elasticity using the inverse of the price impact:

$$\frac{1}{\partial p_j / \partial E_j} = -\frac{1}{\gamma\sigma_\epsilon^2}$$

- As payoffs are independent, they overlap in all states! ($\boldsymbol{YY}^\top$ is not diagonal)
- Why doesn't the trilemma apply? $\boldsymbol{V}$ is not diagonal in this economy

Is No Spillover Necessary for Elasticity Identification?

- Now consider the case where $\sigma_F^2 > 0$, so there is a spillover effect

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- With $-\frac{1}{\gamma(\sigma_\epsilon^2 + \sigma_F^2)}$ and $-\gamma\sigma_\epsilon^2$, we can back out the elasticity $\frac{\partial a_1}{\partial p_1}$

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- **An analogy from factor models**:
  - An impossibility theorem: We can't solve portfolio choice problems because we can't accurately estimate an $N \times N$ covariance matrix Σ

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  - Advantage: Flexible and model-agnostic
  - Disadvantage: Too much flexibility! Need more theory-guided structures
- If a more structural model can fit better, just estimate the structural parameters
  - Current state: relatively few models can be easily taken to data
  - Theoretical contributions are highly valued!
- Huge synergy between theories and empirics. Looking forward to more theoretical discussions!

References I